

Compare and Contrast of Option Decay Functions

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Practitioner's Abstract: *The objective of this study is to provide an initial examination of the observed decay paths for option premiums for a diverse array of option products: corn, soybeans, crude oil, gold, and the S&P 500. A review of the literature finds only limited analysis and therefore limited information on the attributes of the option decay function. Corn and soybeans are agricultural commodities with a defined harvest, crude oil is an energy source and industrial input with continuous production, gold is a precious metal with continuous production, and the S&P 500 is an equity index of stock values. The time period analyzed is the 2007-2012 crop years, a period of volatile commodity and equity prices and increased use of options. As expected from the theoretical models of option prices, the option decay function is nonlinear, with the decay becoming more rapid as expiration of the option contract approaches. Relatively little difference has been found in the decay function across different contract maturities for the same product, but differences in decay functions have been found across products. The different rates of decay and the differences across products suggest that management of cost resulting from option decay may be an important consideration in using options to manage risk and earn profit. Last, even though the time value of an option decays with a known certainty, it does not result in trading returns to a simple, sell-and-hold strategy for a short option position.*

Keywords: futures, option, decay, at-the-money, corn, soybeans, S&P 500, gold, crude oil

Introduction

Option contracts are commonly used to manage risk and earn profit. Buying an option requires the payment of an option premium to the seller of the option. The option premium declines over time as expiration of an option nears. Understanding this option decay function is important for the effective use of options by farmers, businesses, and investors alike.

The pricing of option contracts was first explained by Fischer Black and Myron Scholes. They concluded that the actual option price where the option is bought or sold differs from that which is predicted by the valuation formula (for discussion, see Black and Scholes, 1973). The economists also found that this difference is greater for low-risk stocks than for high-risk stocks. Robert Merton furthered option pricing in his own research, concluding that the Black–Scholes Model can be applied successfully.

One particular point of interest regarding options is the aforementioned option decay function, which is expressed in the Black-Scholes Model. This decay path was examined by King and Zulauf using December corn and November soybean at-the-money option contracts, as there was no prior literature on this topic. It was found that the cost associated with option premium decay is relatively small when buying a December corn or November soybean option, dependent on the option position being closed out before mid-to-late June. After this point, the decay function accelerates, resulting in higher costs to buyers of such products.

When examining the literature, there is a lack of research conducted that compares the contracts used by King and Zulauf with other corn and soybean option contracts. Moreover, there is little to no literature comparing corn and soybean options with other products. Using data from the Commodity Research Bureau, this study compares corn and soybean option contracts across contract maturities, and then compares these contracts with gold, crude oil, and the S&P 500 during the post-commodity price run-up period containing the 2007-2012 crop years for corn and soybeans. A crop year for these two commodities begins September 1 and concludes August 31.

The rest of this article is organized as follows. First, the methods used to examine the data will be discussed. A comparison of the time path of at-the-money option time value as a percent of option strike price for March, May, July, September, and December corn is examined in the next section starting at 240 days until expiration. This is followed by the same comparison for January, March, May, July, and November soybeans from 180 days to expiration. Next, the contracts for each commodity are combined, and a comparison is made between corn and soybeans. Then, the time path of the rate of decay is examined for corn and soybeans from 180 days to expiration. This is followed by a comparison of the aforementioned ratio for corn, soybeans, gold, crude oil, and the S&P 500 from 64 days to expiration, as well as a comparison of the time path of decay functions for these five products. Gross return will be evaluated for call and put options across all products assuming a sell-and-hold strategy, and, lastly, the conclusions and implications will be discussed.

Conceptual Discussion

In order to compare the time value of options, one must first divide the option into intrinsic and time (extrinsic) value. Intrinsic value is the value of the option if the holder of the option exercises the option. Intrinsic value is calculated as:

- (1) Intrinsic Value of a Call: if $\text{futures price} - \text{strike price} > 0$, $\text{intrinsic value} = (\text{futures price} - \text{strike price})$; if $\text{futures price} - \text{strike price} \leq 0$, $\text{intrinsic value} = 0$
- (2) Intrinsic Value of a Put: if $\text{strike price} - \text{futures price} > 0$, $\text{intrinsic value} = (\text{strike price} - \text{futures price})$; if $\text{strike price} - \text{futures price} \leq 0$, $\text{intrinsic value} = 0$

In essence, Equation 1 is stating that as a buyer of a call option, the option has intrinsic value if the futures price is greater than the strike price, as it is cheaper to buy the product at the strike price than at the futures price (market price). Likewise, Equation 2 states that a put option will have intrinsic value given that the strike price exceeds the futures price, as it is more lucrative for the option holder to sell the product at the strike price as opposed to the futures price.

Once intrinsic value has been calculated, the remaining value is considered the time value of said option. In order to compare the time value of each option across contracts, at-the-money time value of each contract was taken as a percent of the option strike price. It was calculated as:

- (3)
$$[(\text{Product Premium} - \text{Product Intrinsic Value}) / \text{Product Strike Price}]$$

This method was repeated over option call and put contracts for corn, soybeans, gold, crude oil, and S&P 500 from the 2007 – 2012 crop years. Seeing no tangible difference due to the concept of put-call parity, the call and put contract ratios for each trading day across all contracts were averaged, to create an average option contract for each respective product. Taking into consideration observed trading volume for recent trading periods, and assuming that options are more commonly traded at present day than in earlier time periods, it was determined to gauge corn from 240 days to expiration, soybeans from 180 days to expiration, and gold, crude oil, and the S&P 500 from 64 days until expiration.

In addition to calculating the aforementioned ratio, the decay of each option's time value was calculated over the same time frame and days to expiration. The calculation is as follows:

- (4)
$$[(\text{Average Time Value Ratio at day } t / \text{Average Time Value Ratio at day } t+1) - 1]$$

Similar to the time value ratio, the decay functions for each product were calculated across all contract maturities for a product-specific number of days to maturity.

In order to test whether products differed within or amongst themselves, a simple, two-mean statistical test was conducted across all days of trading for each product. The calculation is:

- (5)
$$\frac{\{[(\text{mean value for product } x) - (\text{mean value for product } y) - 0] / [\text{square root of } ((\text{number of observations on product } y \times \text{variance of product } y\text{'s observations}) + (\text{number of observations on product } x \times \text{variance of product } x\text{'s observations}))]\} \times \sqrt{\{[(\text{number of observations of product } y \times \text{number of observations of product } x) \times (\text{number of observations of product } y + \text{number of observations of product } x - 2)] / (\text{number of observations of product } y + \text{number of observations of product } x)\}}\}}$$

Once computed, these values were then compared with appropriate two-tailed t-table values given comparison-specific degrees of freedom at the 95% confidence interval. The number of daily instances considered significant in each comparison was then taken as a percentage of possible observations.

Lastly, gross trading return was calculated assuming a sell-and-hold strategy for both calls and puts at 180, 64, and 10 days to expiration for corn and soybeans, as well as at 64 and 10 days to expiration for gold, crude oil, and the S&P 500. This was conducted for each individual contract maturity across all products, and then averaged to find a mean gross trading return for a product call and a product put. This was calculated as follows:

- (6) Gross Call Trading Return: if $[\text{premium at day } t - (\text{futures price at expiration} - \text{strike price at day } t)] < (\text{premium at day } t)$, gross call trading return = $[\text{premium at day } t - (\text{futures price at expiration} - \text{strike price at day } t)]$; if $[\text{premium at day } t - (\text{futures price at expiration} - \text{strike price at day } t)] \geq (\text{premium at day } t)$, gross call trading return = (premium at day t)
- (7) Gross Put Trading Return: if $[\text{premium at day } t - (\text{strike price at day } t - \text{futures price at expiration})] < (\text{premium at day } t)$, gross put trading return = $[\text{premium at day } t - (\text{strike price at day } t - \text{futures price at expiration})]$; if $[\text{premium at day } t - (\text{strike price at day } t - \text{futures price at expiration})] \geq (\text{premium at day } t)$, gross put trading return = (premium at day t)

Essentially, Equation 6 is stating that, as the writer of a call option, the gross trading return will equal the premium received for writing the option, less the difference of the futures price and the strike price. If this results in an outcome where the projected returned value is more than the original value of the premium (the price received for the option), then the option holder will let the option expire worthless, and the gain would simply be the premium, as the option holder would lose more than just the premium if the option was exercised. If the projected return is less than the premium, then the option holder will exercise the option as a means to either reduce lost premium or make a gain, resulting in lost returns for the writer, who simply collects the premium. Equation 7, meanwhile, takes a similar, yet mirrored approach, as this relates to a put option instead of a call. A put holder will only exercise the option if the return value of the strike price at day t less futures price at expiration and premium paid reduces the loss that would be incurred by simply allowing the option to expire and incurring only the loss of the premium paid, or if the return value results in a gain. The portion that is a return on the premium paid for the holder is a loss to the put writer. These returns were statistically tested for significance using a traditional, one-tailed t-test at 95% confidence.

Methods and Data

As stated prior, this study encompasses the crop years of 2007-2012, or the period of time post-agricultural commodity price run-ups in 2006. While the years covered are constant, the contract maturities for each contract differ. Potential for future research exists to compare this chosen period for corn and soybeans with the crop years of 2000-2005, or the time frame prior to the agricultural price run-up. In doing so, one could potentially determine if differences occur in option behavior across time periods.

For this specific study, corn, soybeans, gold, crude oil, and the S&P 500 were chosen in order to compare option decay across different types of products. Corn and soybeans were chosen as they represent the two largest agricultural commodities, and have a distinguished growing season and harvest. Gold is a precious metal, crude oil is an energy product, and the S&P 500 is an index of stocks.

For corn, the months of March, May, July, September and December are included. Each contract's beginning value differed, as would be expected in an actively traded market. Additionally, the last day with which each contract was traded varied within each contract month across years, with a six-day variance for all contracts. Also, the beginning day of trading varied. Thus, after taking these factors along with trading volume into account, the authors decided to standardize the number of trading days for each corn option to 240.

For soybeans, the months of January, March, May, July, and November are included. Similar to corn, each contract's beginning value varied, as did the final trading day and beginning of trading. Given such, in addition to volume constraints, the number of trading days for each soybean contract was standardized to 180.

Gold, crude oil, and the S&P 500 have a similar storyline. Gold includes contract maturities for February, April, June, August, October and December. Meanwhile, crude oil has contracts representing all 12 calendar months, while the S&P 500 has March, June, September and December traded. All contracts have differing end dates, beginning dates, and beginning values. Thus, again taking these factors and trading volume into consideration, 64 days was decided upon as the standardized days to expiration for these three products.

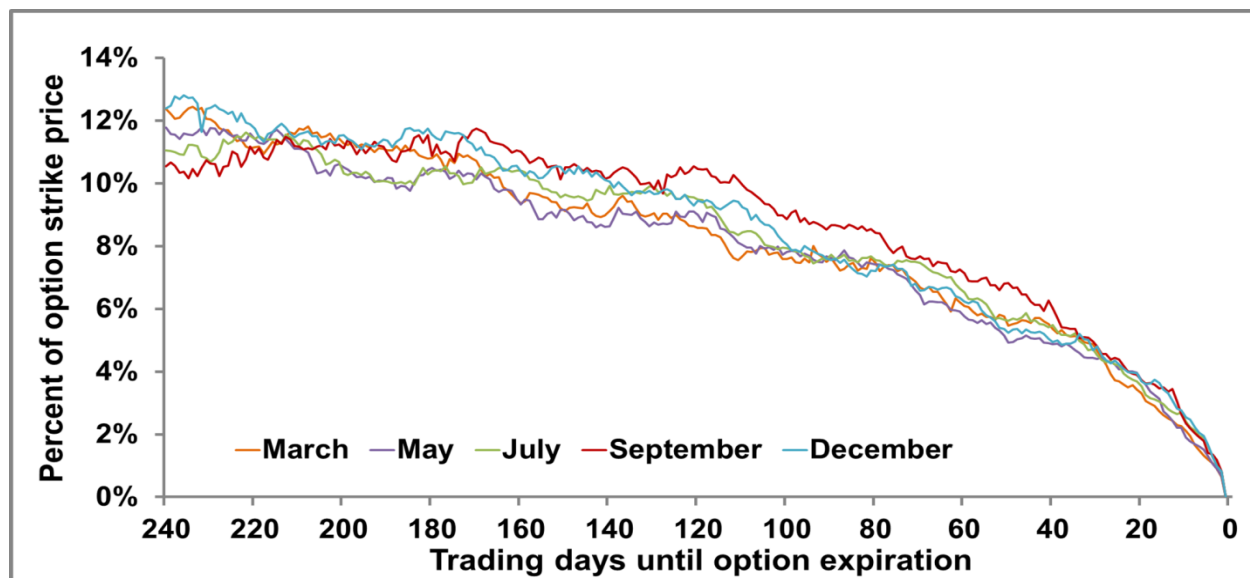
Each option contract is traded at specified strike prices, which vary by a given amount. For example, November soybean calls and puts trade at strike prices of \$12.00, \$12.10, \$12.20, etc, trading at 10 cents per bushel increments around the at-the-money price, or the strike price closest to the settlement price for the underlying futures contract. Meanwhile, corn option contracts trade at 5 cent per bushel intervals. Crude oil trades at 50 cent per barrel increments for the first twenty strike prices above and below at-the-money and \$2.50 per barrel thereafter. Gold, at all times, has at least forty, \$5 strike prices above and below the at-the money strike price, ten, \$10 increments above and below the \$5 increments, and eight, \$25 strike prices above and below the \$10 increments. The S&P 500 trades at 25 point intervals within 50% of the previous day's settlement price of the underlying futures, 10 point intervals within 20%, and 5 point intervals once the option becomes the second nearest contract within 10%. All option specifications can be found on the Chicago Mercantile Exchange website

Results — Option Decay

Time value as a percent of the corn option strike price for March, May, July, September, and December contract expirations were calculated for each crop year from 240 days to expiration. Each month was then averaged across years to construct an average corn contract for each contract maturity.

At first glance at Figure 1, each month appears similar in level and pattern of decline. Each month begins in the 11%-13% range, and begins to track downward, with no month consistently higher or lower throughout the life of the option. A statistical test of the difference between the means of the contract expiration months confirms this, as there were no statistical differences found at the 95% confidence level. In fact, the test suggests that only 1.39% of all observations are significantly different at 95% confidence. This simple, crude test suggests that the ratio of the time value adjusted premium to the strike price is the same for all contract months.

Figure 1: Time path of at-the-money option time value as a percent of option strike price, Yellow Corn, U.S., 2007-2012



In Figure 2, time value as a percent of the soybean option strike price for January, March, May, July, and November contract expirations were calculated for each crop year from 180 days to expiration. Each month was then averaged across years to construct an average soybean contract for each contract maturity. Similar to corn, at first glance, each month seems similar in level and pattern of decline. Each month begins in the 8% to 10% range, and begins to track downward, again with no month consistently on the high or low end throughout the life of the option. A statistical test of the difference between the means of the contract expiration months confirms this, as there were no statistical differences found at the 95% confidence level. In fact, the test suggests that 0% of observations are significantly different at 95% confidence. Consistent with

corn, this simple, crude test suggests that the ratio of the time value adjusted premium to the strike price is the same for all contract months.

Figure 2: Time path of at-the-money option time value as a percent of option strike price, Soybeans, U.S., 2007-2012

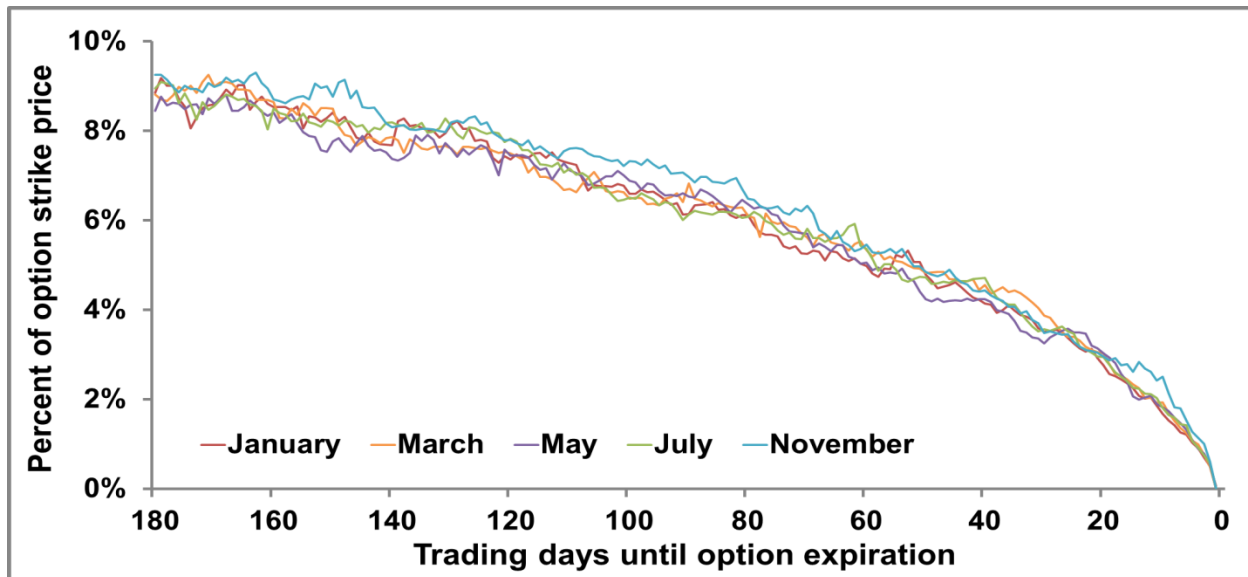
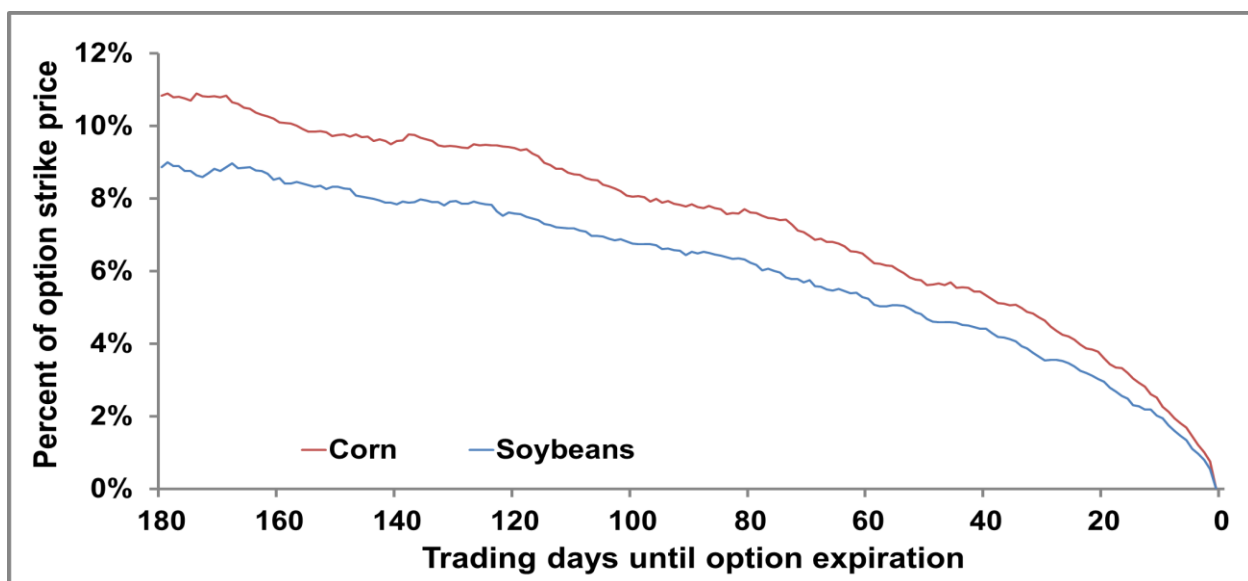


Figure 3: Time path of at-the-money option time value as percent of option strike price, Average Corn vs. Soybeans, U.S., 2007-2012

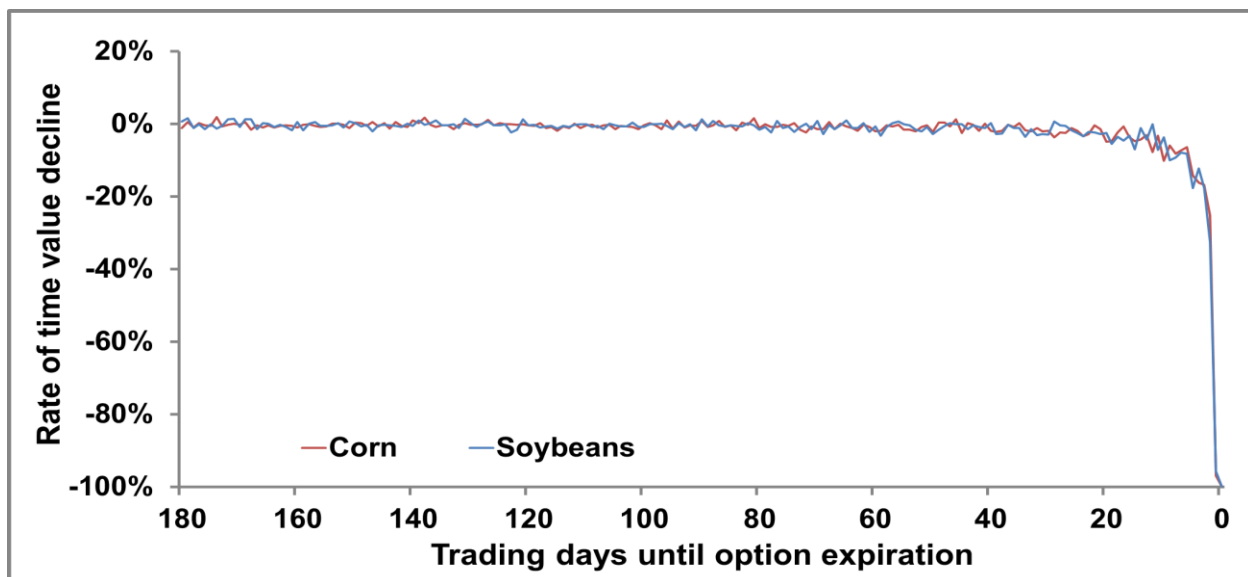


Seeing as there were no statistical differences at 95% confidence for both corn and soybeans, the contracts were averaged into a single corn and single soybean option contract. Because soybeans were being measured over 180 days, corn observations were cut to 180 days to expiration as well. When comparing the time value as a percent of strike price for corn versus soybeans in Figure 3, it appears that corn is in fact higher than soybeans.

After statistically testing, it was found that corn was in fact statistically higher than soybeans starting at 180 days to expiration. Knowing this occurrence is potentially beneficial for traders looking to enter the option market, as this is basically saying that corn options are more volatile than soybean options at each given day to expiration.

The next step in comparing corn versus soybean options is to explore their respective decay functions. It was found that rate of daily decay in corn and soybean option time value is strikingly similar from 180 trading days until expiration. This is displayed in Figure 4. As expected from option theory, rate of decay accelerates as expiration nears. In this case, it was observed beginning at roughly 14 days to expiration that decay began to occur exponentially. Prior to this time, the option did not substantially decay.

Figure 4: Time path of at-the-money option rate of time value decay, Average Corn vs. Soybeans, U.S., 2007-2012

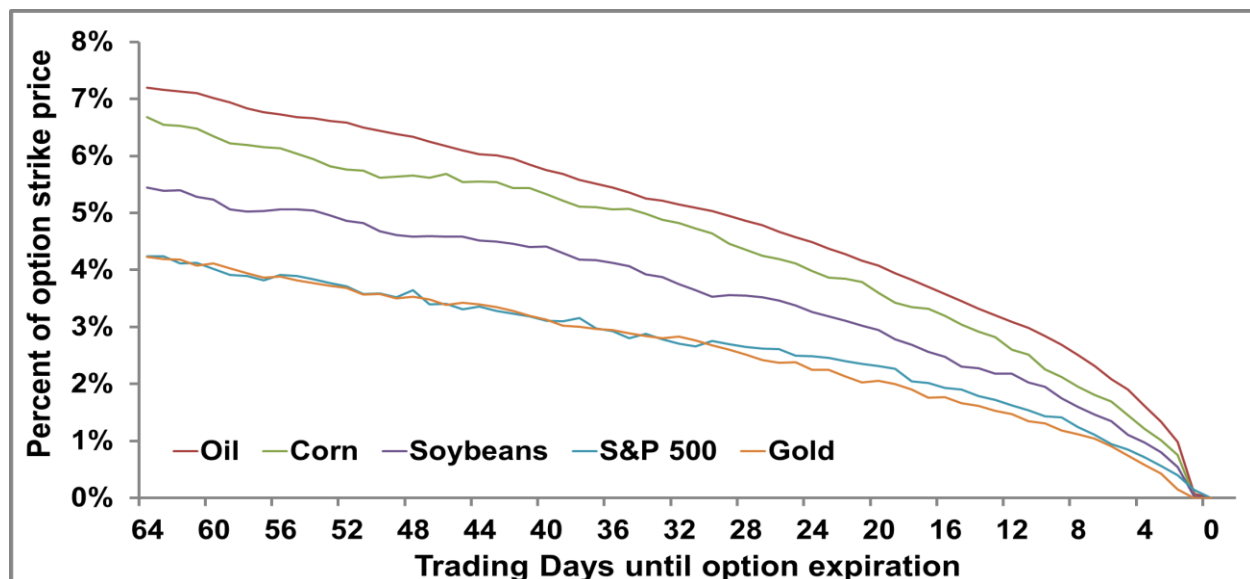


In order to test for differences between the two functions, the statistical test was performed. No significant difference was detected between rates of decay at the 95% confidence level. This result was, in a sense, surprising, as each crop has different characteristics, as well as slightly different growing seasons. However, this is useful for farmers and market players looking to manage risk by holding an option, as the at-the-money option maintains its intrinsic value until about two weeks to expiration.

Figure 5 depicts the path of time value as a percentage of option strike price for all products – corn, soybeans, crude oil, gold, and the S&P 500. This comparison was performed beginning at 64 days to expiration. This is due to a perceived lack of trading volume of the non-agricultural commodities beyond 64 days, or roughly two months.

Over the last 64 days of option trading, crude oil has the highest ratio of time value to strike price, with corn being the next highest. Gold and the S&P 500 have the lowest ratios, while soybeans are in the middle. Statistical differences at the 95% confidence level were found between nearly every product pair except corn relative to crude oil, and the S&P 500 relative to gold. Thus, nearly every product, excluding corn relative to crude oil and the S&P 500 relative to gold, has different levels of volatility. These findings are not altogether shocking, as the products inherently have different characteristics.

Figure 5: Time path of at-the-money option time value as percent of option strike price, All Products, U.S., 2007-2012

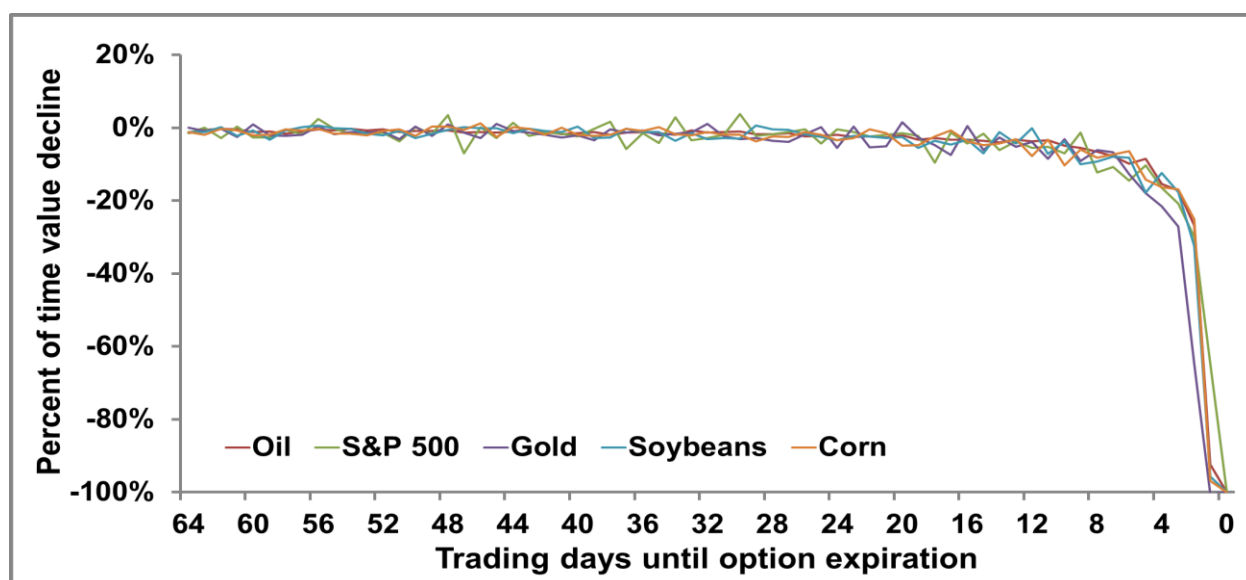


Once again, the next step in comparing these five different products' options is to explore their respective decay functions. This is shown in Figure 6. As expected from option theory, rate of decay accelerates as expiration nears. It was found that rate of daily decay in at-the-money option time value is strikingly similar from 64 trading days until expiration for all products. As can be seen in Figure 6, the decay functions begin to get more unstable as the option approaches 14 days to expiration, and takes off at around seven days remaining. Prior to this time, the option did not substantially show decay in time value.

Again, in order to test for differences between the five functions, the two-mean statistical test was performed. No significant difference was detected between rates of decay at the 95% confidence level. This result was also in a sense surprising as, building from the corn and soybean discussion around Figure 4, each product has vastly different characteristics. However,

this knowledge of the characteristics of decay functions is potentially extremely useful for farmers and market players looking to manage risk by holding an option, as the at-the-money option maintains its time value until about 14 days to expiration, before it begins to exponentially decay around the seven-day mark.

Figure 6: Time path of at-the-money option rate of time value decay, All Products, U.S., 2007-2012



Results — Option Trading Returns

Given the existence of the decay in option time value, an issue that arises is whether this decay results in gains for sellers of options and losses for buyers of options since the decay in time value means the value of premium will decline, resulting in gains for option shorts and losses for option longs. To examine this potential implication, gross trading results were calculated for an option sell-and-hold trading strategy initiated at 10, 64, and 180 days prior to expiration. The latter two times to expiration were used throughout this study. The 10 day period coincides with the period of most rapid decay in the option time value for all 5 assets (see Figure 6).

Average gross trading return over all contracts for an asset for the 2007-2012 period are presented in Table 1. Average returns are calculated for both calls and puts. Eleven of the 24 averages are greater than zero. As expected, the average return for a short put and the average return for a short call usually had opposite signs — when one was positive, the other was negative. Only two of the average returns are statistically greater than zero at the 95% confidence level using a one-tail test: a gold put at 64 days-to-expiration and a S&P 500 put at 10 days-to-expiration. A one tail test is used since option decay works in favor of the option seller.

In conclusion, when taken as a group, the trading results in Table 1 implies that, even though the time value of an option decays with a known certainty, it does not result in trading returns to a short option position.

Table 1: Average trading returns for sell and hold strategy: at-the-money call options (at-the-money put options), US dollars/unit, 2007-2012

<u>Product</u>	<u>Units</u>	<u>180 Days to Expiration</u>	<u>64 Days to Expiration</u>	<u>10 Days to Expiration</u>
Corn	\$/bushel	-\$0.26 (\$0.04)	-\$0.02 (-\$0.02)	\$0.01 (\$0.00)
Soybeans	\$/bushel	-\$0.61 (\$0.30)	-\$0.22 (\$0.14)	-\$0.04 (\$0.10)
Crude Oil	\$/barrel	-	\$0.005 (-\$0.004)	\$0.004 (-\$0.003)
Gold	\$/troy ounce	-	-\$0.11 (\$0.23)	-\$0.05 (-\$0.02)
S&P 500	\$/index point	-	\$0.03 (\$0.10)	-\$0.10 (\$0.07)

Conclusions and Implications

No consistent difference is found in option time value across the different contract maturities for corn or soybeans, despite the different relationship between contract expiration and the growing season for each crop. When comparing the ratio of time value relative to strike price for an average corn option versus an average soybean option, however, corn is statistically higher than soybeans, and thus more volatile. No consistent difference is found in the rate of decay in option time value for corn, soybeans, crude oil, gold, and the S&P 500 over the observed times to expiration. The only statistical difference across the five option assets is the level of time value as a percent of the option premium for all but two pairwise comparisons – corn relative to crude oil, and the S&P 500 relative to gold.

Despite the known decay in option time value, an analysis of a simple, sell-and-hold trading strategy reveals no consistent evidence of trading returns to sellers of options. Thus, the known decline in option time value must have been bid into the cost of the premium; in effect, resulting in a lower premium paid by the option buyer and a lower premium received by the option seller. In other words, some benefit must exist that exactly equals the option time value at the time the option position is taken. The identification and explanation of this benefit is not obvious and is an area for further investigation.

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